Use of Fracture Mechanics in Estimating Structural Life and Inspection Intervals

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As structural efficiency of aircraft has increased in the recent past, the Air Force has found it necessary to include, in the newer systems, damage tolerance criteria for fracture critical parts. These criteria are explained briefly, and sample calculations indicating methods for determining component structural life are demonstrated. Implicit in this calculation is the definition of inspection intervals. The second portion of this discussion details some of the assumptions that go into this analysis and their sensitivity. A large portion centers on stress spectrum definition. It is shown that, from one set of occurrence data, alternative spectra may be derived which have substantial differences in life.

Nomenclature

a	= crack length, in.
C,n	= crack growth law parameters
da/dn	= crack propagation rate, in./cycle
$f(a/r)_m$	= modified Bowie correction factor
K	= stress intensity factor, psi in.$^{1/2}$
ΔK	= cyclic stress intensity range
n_z	= vertical load factor, g
r	= hole radius
$\{sec[\pi(a+r)/W]\}^{1/2}$	= finite width correction factor
W	= panel width
α	= proportionality constant, used to account for geometry in stress intensity solution
σ	= far-field stress, psi

Introduction

THE importance of defining a usage stress spectrum for military aircraft and using that spectrum in conjunction with damage tolerance methods to determine the inspection interval throughout the service life of the aircraft cannot be overstated. However, before delving into the details of spectrum definition and its effect on fracture analysis and inspection intervals, it would be advantageous to describe briefly the different methods of airframe damage tolerance defined design. The Air Force recently has instituted an Airframe Structural Integrity Program (ASIP).[1] ASIP is a general plan, the purpose of which is to assure structural safety of U.S. Air Force aircraft, and it references a set of specifications, defining specific requirements. One of these specifications, MIL-A-83444,[2] relates to the damage tolerance criterion. This specification defines the types of structural designs.

Three structural designs, shown in Fig. 1, are 1) slow crack growth (design concepts where flaws cannot be allowed to attain the critical size required for unstable crack propagation due to monolithic construction), 2) fail-safe independent (structure, where, by design, it is unlikely that a common source of cracking exists in more than a single load path at one location due to the nature of the assembly or manufacturing procedures), and 3) fail-safe dependent (structure, where, by design, a common source of cracking exists in adjacent load paths at one location due to the nature of the assembly or manufacturing procedures). For slow crack growth, the assumed initial damage introduced during production is a 0.05-in. crack at a fastener hole, or a 0.25-in. semicircular crack away from a hole. For both types of fail-safe structure, the assumed initial damage is a 0.02-in. crack at a fastener hole.

This by no means completes the criteria. As an example, for fail-safe design, there is a remaining structure policy. This policy states that, subsequent to primary load path failure, the remaining structure must fulfill several criteria which include initial damage and inspectability. Other important factors are 1) several flaw geometries which include surface and through-the-thickness cracks away from holes, 2) life for uninspectable structure must be shown to be two lifetimes, both analytically and experimentally, and 3) the structure must be capable of tolerating flaws of designated lengths for designated periods of time. Also included in MIL-A-83444 is the flaw size which must be assumed after the aircraft is inspected by maintenance personnel during major overhaul, the necessary residual strength that the component must possess with damage present, and the length of time that the residual strength must be maintained.

With this introduction, it now is possible to examine some of the important factors used when applying fracture mechanics to aircraft structures. This will be done by describing how and what types of spectra can be derived and some of the idiosyncrasies associated with each. There then will be a short discussion describing an interpretation of inspection interval and, finally, a few examples of the type of analysis which confronts fracture mechanists on a daily basis.

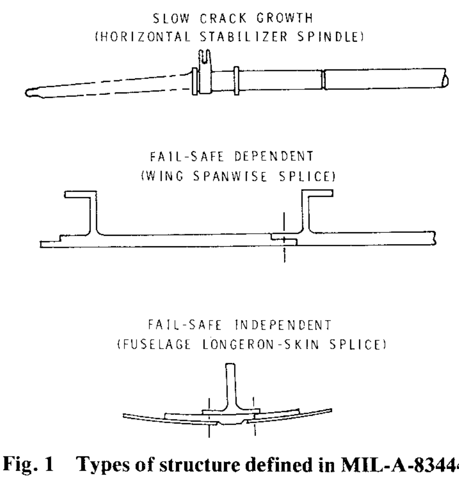

Fig. 1 Types of structure defined in MIL-A-83444.

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Spectrum Derivation

In deriving the loading spectrum, the normal procedure is first to develop a load exceedance curve. This curve, shown, in Fig. 2, is a plot of stress (or vertical load factor n_z or bending moment) vs cumulative number of exceedances (for a given time interval); i.e., it shows the total number of times a certain stress will be exceeded during the lifetime of the aircraft. This information can be obtained from a number of sources: MIL-A-8866A,[3] recorded flight data from either the actual aircraft or similar ones, etc.

For a given exceedance curve, several different types of spectra can be developed. These spectra fall into two basic categories: block and flight-by-flight. A block spectrum is the simplest to derive, test, and analyze. However, the flight-by-flight spectrum certainly is more representative of actual aircraft usage.

A block spectrum consists of a group of load cycles which are ordered on the basis of magnitude, e.g., lowest to highest, but with no thought given to the actual order in which these loads would occur on the real aircraft. Simply speaking, a block spectrum would be developed by dividing the exceedance curve into a number of stress intervals, and then determining the number of occurrences within each stress interval. Since the exceedance curve is based on a certain number of flight hours, the block spectrum derived represents the same number of flight hours. If the block size is less than the life of the airplane, it is simply repeated an appropriate number of times. Probably the most important consideration, after one has determined the block size and number of occurrences of each stress in that block, is the order in which the stress will occur. A number of options are available, and the resulting life can vary greatly. The most common procedures are to order the loads from lowest to highest, highest to lowest, or low to high to low. These are shown in Fig. 3. Several studies have shown that a low-high spectrum results in the shortest life, the high-low spectrum in the longest life, and the low-high-low spectrum is between the two.[4-6] The large differences in life are caused by crack growth retardation.

Retardation, a physical phenomenon which causes the crack growth rate to decrease after application of a high load, [7,8] is due to the compressive force field which is induced around the cracked area. As the crack grows, the crack tip moves out of the compressive zone and the retarding effect will decay. This explains the life differences shown in Fig. 3. The high-low spectrum causes much retardation, whereas in the low-high spectrum this overload effect is nonexistent. The low-high-low spectrum is the most widely accepted formulation since it falls between the two extremes and, thus, is considered to be the most "reasonable."

The other common type of spectrum, shown in Fig. 3, is a flight-by-flight spectrum. As the name implies, this type of spectrum consists of a series of rational flights which make up the lifetime of the aircraft. By rational we mean that the first loads in the flight are taxi and takeoff loads followed by flight loads, followed by landing loads. This differs from a block spectrum where loads normally are grouped just based on magnitude, i.e., low-high, with no real thought given to actual order of occurrence. Another large difference between flight-by-flight and block spectra is the relative time covered by a flight compared to a block. A flight normally would last on the order of, perhaps, two hours, whereas a block can last hundreds of hours. It has been shown that the shorter the block, the closer the answer agrees with flight-by-flight results.[4-6] It is apparent from this discussion that a spectrum can take on aspects of both a flight-by-flight and a block spectrum. For example, a block spectrum with very short blocks approaches a flight-by-flight spectrum or, in a flight-by-flight spectrum, there can be a small number of different flights or missions which are repeated in the same order to simulate one lifetime. The most representative flight-by-flight spectrum is one where the type of mission to be flown, flight duration, and order of occurrences of loads within the flight are selected completely randomly. With the advent of high-speed computers, this type of problem can be handled fairly easily.

Another problem presents itself when defining a ground-air-ground (GAG) cycle. The GAG cycle is the long-period, high-amplitude cycle which accounts for the change in mean stress from the ground to the air. Originally, a GAG cycle was

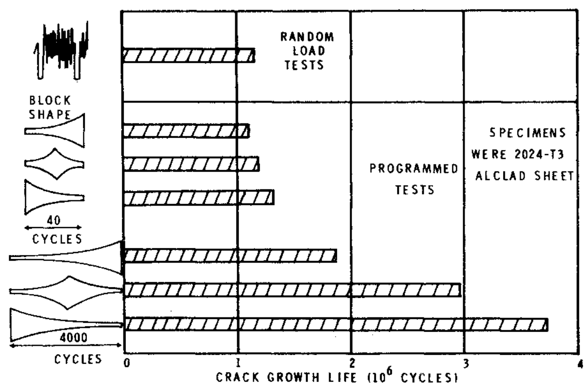


Fig. 3 Effect of block size and load sequence on fatique life.

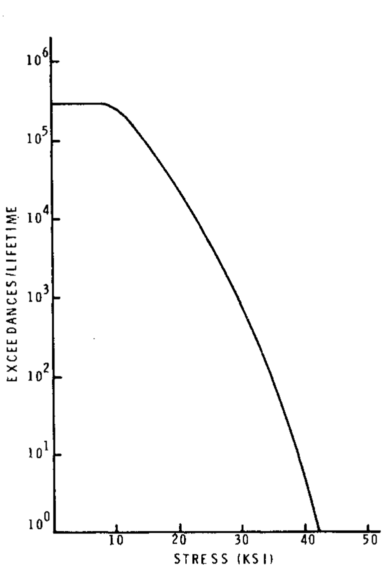


Fig. 2 Typical stress-exceedance curve.

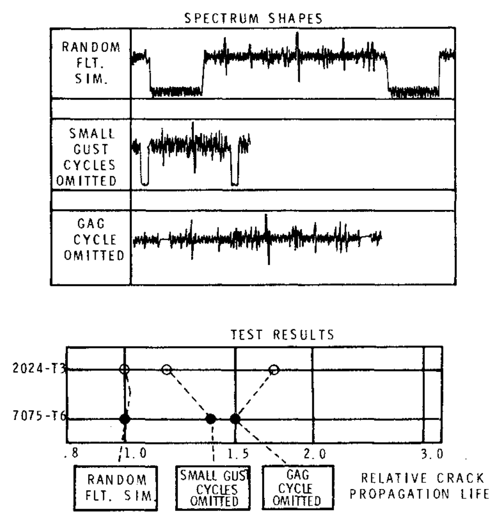


Fig. 4 Effect of omitting certain types of cycles from a flight-by-flight spectrum.

defined as the excursion from the mean load on the ground to the mean load in the air. However, in trying to predict failures which occurred in fleet usage, or in flight-by-flight test, a peak-to-peak definition of the ground-air-ground cycle gave the best results and, therefore, is the one which should be used. For a cargo aircraft, a peak-to-peak GAG cycle may be two or three times more damaging than a mean-to-mean GAG definition, and, as the GAG cycle can account for up to 90% of the total damage, the correct definition is extremely important. Figure 4 illustrates this to some extent.[9] In a fighter aircraft this is less important as there are a large number of maneuver excursions which are of the same magnitude as the GAG cycle. For a fighter-type spectrum the important thing to consider is negative maneuver loads. As mentioned earlier, negative stresses increase the crack growth rate by reducing the retardation effects. Because a fighter has such a large number of high positive loads which causes much retardation, it is of paramount importance to include any effects which would lessen the retardation. Test results indicate that a factor of 2 may exist in life, dependent upon the inclusion or exclusion of negative stresses.[10,11] Unfortunately, crack growth computer programs, e.g., CRACKS,[12] EFFGRO,[13] etc., do not have the capability for handling negative loads and automatically set them equal to zero. Thus, it can be seen that crack growth programs cannot predict the life, always adequately especially when complex loading spectra include the retarding effects of high loads and their eradication by negative loads.[14] This is why Air Force specifications require tests for design substantiation.

Inspection Interval Determination

To determine a repeating inspection interval, it is necessary to perform a fracture mechanics calculation. This is required in determining the life. In order to do this two equations must be defined. They are[15,16]

$$K=\sigma(\pi a)^{1/2}\alpha \tag{1}$$

$$da/dn=C\Delta K^n \tag{2}$$

Equation (2) is not universal; however, for ease of calculation, it will be used. A fracture mechanics analysis may be done in the following way. For the part in question, the stress is determined analytically. The crack growth characteristics including C and n are obtained by experiment. The stress intensity factor K is calculated. Equation (2) then is integrated, numerically, and a "crack length vs life" curve is obtained from which inspection intervals can be determined.

The question is, what is an inspection interval? If a structure has been designed such that a particular component cannot be inspected, Air Force specifications require that, with the given initial flaw size, that component must remain structurally adequate for two design lifetimes. If the structure is designed such that the component is inspectable, it must remain structurally adequate for two times the chosen inspection interval.

Assume that a particular component shows safe crack growth for 1000 flight hours with a given initial flaw size of a 0.05-in. through-crack located at a fastener hole. Using a safety factor of 2, that part would be inspected every 500 hr. If no flaw were seen after 500 hr, a 0.05-in. flaw again would be assumed, and another inspection would occur after 500 additional service hours. If a flaw were seen, its length would be measured and its useful life could be determined from a crack length vs time curve. If it is inadequate, the part would be replaced immediately. Assume, however, that after 500 hr there was no flaw and, again, after 1000 hr there was no flaw. The part need not be replaced; it may be used as long as the flaw size remains at a reasonable level. In a similar situation, classic fatigue theory might require that the part be replaced after a given number of hours have elapsed. This is only one advantage of fracture mechanics. The structural "hot spots" may be identified and observed.

Sample Calculations

Lug

The first example discussed is a lug. The method shown below may be considered as a classical engineering approach in problem solving. The geometry, along with the loading, is shown in Fig. 5. The stress intensity solution for an infinitely wide center-cracked plate is[15] $K=\sigma(\pi a)^{1/2}$. As there is a hole from which the crack emanates, it is necessary to use the Bowie factor[17] which accounts for the stress concentration of an open hole. As the hole is loaded, the Bowie factor should be modified. That information, the lug loading stress concentration factor, is obtainable from Heywood.[18] Figure 6 compares Bowie's factor with the modified factor. It also is noted that the lug has finite boundaries. Therefore it is necessary to use the Isida correction factor.[19] The final equation is shown below:

$$K=\sigma\{\pi a\sec[\pi(a+r)/W]\}^{1/2}f(a/r)_m \tag{3}$$

Using this equation for K, and the proper spectrum, substituting Eq. (3) into Eq. (2), a 6000-cycle life is obtained after

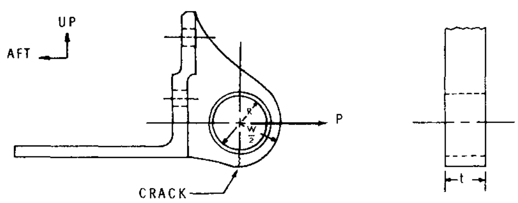


Fig. 5 Typical lug geometry.

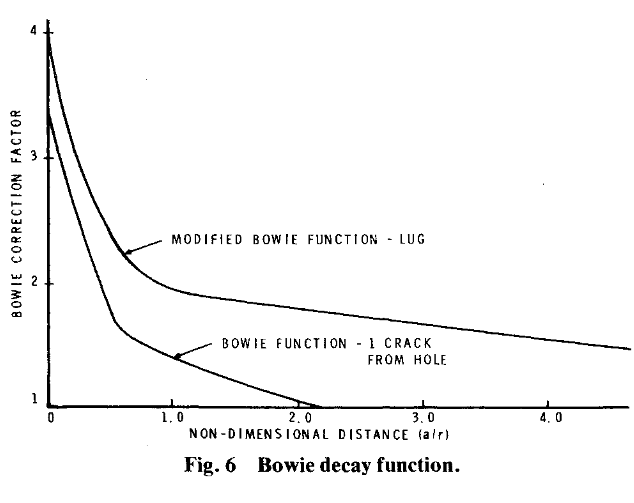
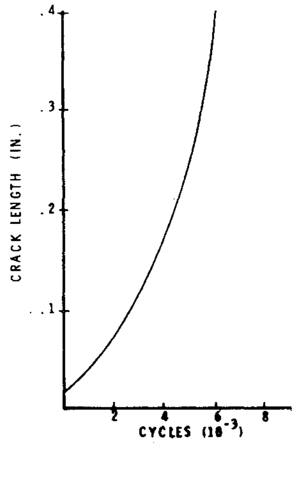


Fig. 6 Bowie decay function.

Fig. 7 Crack growth curve for lug.



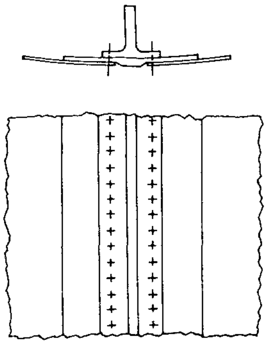


Fig. 8 Longeron geometry.

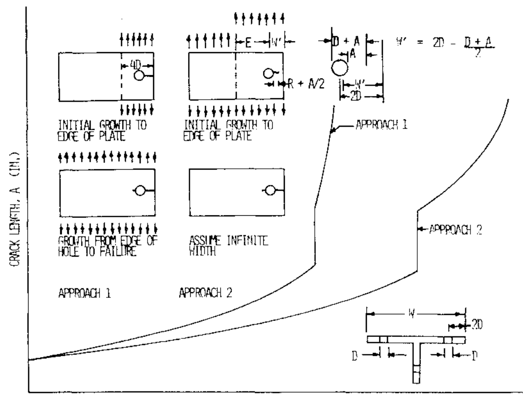


Fig. 9 Typical longeron or spar cap analysis approaches.

numerical integration. The resulting crack-growth vs life curve is shown in Fig. 7. To determine the inspection period, one merely divides the expected life by a factor of 2. As this part is removable, it can be inspected at a major overhaul. The flaw size which can be found is assumed to be a 0.02-in. through-crack. After 3000 cycles, the lug will be inspected to determine if a crack exists. If it does not, the lug will be placed back into service for 3000 more cycles before reinspection occurs. If a crack were missed, a second inspection prior to analytical failure could locate the flaw prior to service failure.

Longeron

The second example is that of a longeron. The geometry of the longeron, shown in Fig. 8, is a typical airframe structural component requiring a damage tolerance analysis. To perform this type of analysis, it is necessary to assume multiple geometries. The first geometry analyzed is a through-crack emanating from one side of the hole. The crack is assumed to progress from the edge of the hole toward the free edge of the longeron. The crack is not placed at the opposite side of the hole growing toward the stiffener, since MIL-A-83444 requires that "the flaw be placed in the most deleterious position in the structure." After the crack reaches the free edge, it then begins to grow toward the stiffener, and now the structural geometry is akin to a single-edge notch specimen.

There is still one additional factor to be considered. The longeron is fastened to the fuselage skin by a doubler. As they are load-carrying members, albeit not 100%, one would not assume that the entire specimen width is the width of the longeron. It is necessary to take into account additional rigidity of the structure due to the integral stiffener and doubler. The most conservative approach would be to assume a hole in the center of a plate, with the plate width being twice the distance from the center of the hole to the edge of the specimen. After the crack reached the free edge, the crack length would be from the free edge to the opposite side of the hole. Figure 9 shows this geometry.

The approach used was what one may consider a middle ground and also is shown in Fig. 9. The hole containing the flaw was placed in its proper orientation from one free edge. The other side was lengthened to take into effect both the load-carrying capacity of the skin and the increased rigidity due to the integral stiffener. The Isida eccentric width correction factor[20] was used along with the Bowie factor[17] for the crack growing to the free edge. After the crack reached the free edge, it then was assumed to be an edge crack but, again, with the increased width. Crack growth curves for both geometries also are included in Fig. 9. The difference in life is quite remarkable: for this aircraft component, nearly 40%.

As this aircraft is in the design stage, it is very important to verify the analytical procedure used. If the first procedure were used, the analytical life would be less than two lifetimes, necessitating a major inspection. If the life were greater than two lifetimes, inspection would be unnecessary, thus saving much inspection money. Which is correct? Large-scale fatigue crack growth tests will tell the answer.

Conclusions

In conclusion, let us re-emphasize the important concept of defining the inspection interval. First, perhaps the most important thing to consider is proper definition of the aircraft usage so that an accurate loading spectrum can be formulated. If one does not know what the loads are, and when they are applied, the rest of the analysis is totally useless. Second, a reasonably accurate stress analysis is needed, not only to get a good characterization of K, but also, in the case of fail-safe structure, to determine the residual strength available. Lastly, let us touch on nondestructive inspection. This is the area where the most improvement probably can be made. At the present time we are forced to assume that very large cracks can exist after in-service inspection. If we can drive the size of detectable in-service cracks down to a size approaching that which can be found during manufacturing, we can enhance the safety of the system, and save the Air Force and the taxpayers a large amount of inspection dollars.

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